Multiplicity and p_t correlations in AA-interactions at high energies

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Abstract

The theoretical description of the correlations between observables in two separated rapidity intervals in relativistic nuclear collisions is presented. It is shown, that the event-by-event p_t–p_t correlation defined as the correlation between event mean values of transverse momenta of all particles emitted in two different rapidity intervals does not decrease to zero with the increase of the number of strings in contrast with two particle p_t–p_t correlation - the correlation between the transverse momenta of single particles produced in these two rapidity windows.

In the idealized case with the homogeneous string distribution in the transverse plane in the framework of the cellular analog of string fusion model (SFM) the asymptotic of p_t-p_t correlation coefficient is analytically calculated and compared with the results of the Monte-Carlo (MC) calculations fulfilled both in the framework of the original SFM and in the framework of its cellular analog, which enables to control the MC algorithms.

In the case with the realistic nucleon distribution density of colliding nuclei the results of the MC calculations of the p_t–p_t correlation function for minimum bias nuclear collisions at SPS, RHIC and LHC energies are presented and analysed.

1 String fusion model (SFM)

The colour string model [1, 2] originating from Gribov-Regge approach is being widely applied for the description of the soft part of the multiparticle production in hadronic and nuclear interactions at high energies. In this model at first stage of hadronic interaction the formation of the extended objects - the quark-gluon strings - takes place. At second stage the hadronization of these strings produces the observed hadrons. In the original version the strings evolve independently and the observed spectra are just the sum of individual string spectra. However in the case of nuclear collision, with growing energy and atomic number of colliding nuclei, the number of strings grows and one has to take into account the interaction between them.

One of possible approaches to the problem is the colour string fusion model [3]. The model is based on a simple observation that due to final transverse dimensions of strings they inevitably have to start to overlap with the rise of their density in transverse plane. At that the interaction of string colour fields takes place, which changes the process of their fragmentation into hadrons as compared with the fragmentation of independent strings. So we have one more interesting nonlinear phenomenon in nuclear interactions at high energies - the field of physics the investigations in which were initiated by pioneer works of academician A.M. Baldin [4].

It was shown [3, 5, 6] that the string fusion phenomenon considerably damps the charged particle multiplicity and simultaneously increase their mean p_t value as compared with the case of independent strings. In accordance with a general Schwinger idea [7] and the following papers [8, 9] (colour ropes model) two possible versions of string fusion mechanism were suggested.

The first version [5] of the model assumes that the colour fields are summing up only locally in the area of overlaps of strings in the transverse plane. So we will refer to this case as a *local* fusion or *overlaps*. In this case one has

$$\langle n \rangle_k = \mu_0 \frac{S_k}{\sigma_0} \sqrt{k} \qquad\qquad \langle p_t^2 \rangle_k = \overline{p^2}\sqrt{k} \qquad\qquad (1)$$

Here $\langle n \rangle_k$ is the average multiplicity of charged particles originated from the area S_k, where k strings are overlapping, and $\langle p_t^2 \rangle_k$ is the same for their squared transverse momentum. The μ_0 and $\overline{p^2}$ are the average multiplicity and squared transverse momentum of charged particles produced from a decay of one single string, and σ_0 is its transverse area.

In the second version [10] of the model one assumes that the colour fields are summing up globally - over total area of each cluster in the transverse plane - into one average colour field. This case corresponds

to the summing of the source colour charges. We will refer to this case as a *global* fusion or *clusters*. In this case we have

$$\langle n \rangle_{cl} = \mu_0 \frac{S_{cl}}{\sigma_0} \sqrt{k_{cl}} \qquad \langle p_t^2 \rangle_{cl} = \overline{p^2} \sqrt{k_{cl}} \qquad k_{cl} = \frac{N_{cl}^{str} \sigma_0}{S_{cl}} \qquad (2)$$

Here $\langle n \rangle_{cl}$ is the average multiplicity of charged particles originated from the cluster of the area S_{cl} and $\langle p_t^2 \rangle_{cl}$ is the same for their squared transverse momentum. The N_{cl}^{str} is the number of strings forming the cluster.

2   Cellular analog of SFM

To simplify calculations in the case of real nucleus-nucleus collisions a simple cellular model originating from the string fusion model was proposed [11]. In the framework of the cellular analog along with the calculation simplifications the asymptotics of correlation coefficients at large and small string densities can be found analytically in the idealized case with the homogeneous string distribution, which enables to use these asymptotics later for the control of the Monte-Carlo (MC) algorithms (see below).

Two versions of the cellular model as the original SFM can be formulated - with local and global string fusion. In this model we divide all transverse (impact parameter) plane into sells of order of the transverse string size σ_0.

In the version with *local* fusion the assumption of the model is that if the number of strings belonging to the ij-th cell is k_{ij}, then they form higher colour string, which emits in average $\mu_0 \sqrt{k_{ij}}$ particles with mean p_t^2 equal to $\overline{p^2} \sqrt{k_{ij}}$ (compare with (1)). Note that zero "occupation numbers" $k_{ij} = 0$ are also admitted.

In the version with *global* fusion at first we define the neighbour cells as the cells with a common link. Then we define the cluster as the set of neighbour cells with non zero occupation numbers $k_{ij} \neq 0$. After that we can apply the same formulae of the global fusion (2) as in the original SFM, where N_{cl}^{str} is the number of strings in the cluster and S_{cl}/σ_0 is the number of cells in the cluster.

From event to event the number of strings k_{ij} in the ij-th cell will fluctuate around some average value - $\overline{k}_{ij}$. Clear that in the case of real nuclear collisions these average values $\overline{k}_{ij}$ will be different for different cells. They will depend on the position $(\mathbf{s}_{ij})$ of the ij-th cell in the impact parameter plane ($\mathbf{s}$ is two dimensional vector in the transverse plane). In the case of nucleus-nucleus AB-collision at some fixed value of impact parameter $\mathbf{b}$ one can find this *average* local density of primary strings $\overline{k}_{ij}$ in the point $\mathbf{s}_{ij}$ using nuclear profile functions $T_A(\mathbf{s}_{ij} + \mathbf{b}/2)$ and $T_B(\mathbf{s}_{ij} - \mathbf{b}/2)$.

In MC approach knowing the $\overline{k}_{ij}$ one can generate some configuration $C \equiv \{k_{ij}\}$. To get the physical answer for one given event (configuration C) we have to sum the contributions from different cells in accordance with *local* or *global* algorithm (see above), which corresponds to the integration over $\mathbf{s}$ in transverse plane. Then we have to sum over events (over different configurations C). Note that as the event-by-event fluctuations of the impact parameter at a level of a few fermi are inevitable in the experiment one has to include the impact parameter b into definition of configuration $C \equiv \{b, k_{ij}\}$.

3   Long-range correlations

The idea [5, 6, 12] to use the study of long-range correlations in nuclear collisions for observation of the colour string fusion phenomenon based on the consideration that the quark-gluon string is an extended object which fragmentation gives the contribution to wide rapidity range. This can be an origin of the long-range correlations in rapidity space between observables in two different and separated rapidity intervals. Usually in an experiment they choose these two separated rapidity intervals in different hemispheres of the emission of secondary particles one in the forward and another in the backward in the center mass system. So sometimes these long-range rapidity correlations are referred as the forward-backward correlations (FBC).

In principle one can study three types of such long-range correlations:
n-n - the correlation between multiplicities of charged particles in these rapidity intervals,
p_t-p_t - the correlation between transverse momenta in these intervals and

p_t-n - the correlation between the transverse momentum in one rapidity interval and the multiplicity of charged particles in another interval.

Usually to describe these correlations numerically one studies the average value $\langle B \rangle_F$ of one dynamical variable B in the backward rapidity window Δy_B, as a function of another dynamical variable F in the forward rapidity window Δy_F. Here $\langle ... \rangle_F$ denotes averaging over events having a fixed value of the variable F in the forward rapidity window. The $\langle ... \rangle$ denotes averaging over all events. So we find the correlation function $\langle B \rangle_F = f(F)$.

It's naturally then to define the correlation coefficient as the response of $\langle B \rangle_F$ on the variations of the variable F in the vicinity of its average value $\langle F \rangle$. At that useful also to go to the relative variables, i.e. to measure a deviation of F from its average value $\langle F \rangle$ in units of $\langle F \rangle$, and the same for B. So it's reasonable to define a correlation coefficient b_{B-F} for correlation between observables B and F in backward and forward rapidity windows in the following way:

$$b_{B-F} \equiv \frac{\langle F \rangle}{\langle B \rangle} \left. \frac{d\langle B \rangle_F}{dF} \right|_{F=\langle F \rangle} \tag{3}$$

As the dynamical variables we use the multiplicity of charged particles (n), produced in the given event in the given rapidity window, and the *event(!)* mean value of their transverse momentum (p_t), i.e. the sum of the transverse momentum magnitudes of all charged particles, produced in the given event in the given rapidity window (Δy), divided by the number of these particles (n):

$$p_t \equiv \frac{1}{n} \sum_{i=1}^{n} |\mathbf{p}_{ti}|, \qquad \text{where} \qquad y_i \in \Delta y; \qquad i = 1, ..., n. \tag{4}$$

So we can define three correlation coefficients: b_{n-n}, $b_{p_t-p_t}$ and b_{p_t-n}, for example:

$$b_{p_t-p_t} \equiv \frac{\langle p_{tF} \rangle}{\langle p_{tB} \rangle} \left. \frac{d\langle p_{tB} \rangle_{p_{tF}}}{dp_{tF}} \right|_{p_{tF}=\langle p_{tF} \rangle}, \qquad\qquad b_{p_t-n} \equiv \frac{\langle n_F \rangle}{\langle p_{tB} \rangle} \left. \frac{d\langle p_{tB} \rangle_{n_F}}{dn_F} \right|_{n_F=\langle n_F \rangle} \tag{5}$$

The n_B, n_F are the multiplicities and p_{tB}, p_{tF} are the *event* (4) mean transverse momentum of the charged particles, produced in the given event correspondingly in the backward (Δy_B) and forward (Δy_F) rapidity windows. In this paper we concentrate only on the p_t-p_t - correlation. Some other results can be found in [13].

4   Analytical results and MC calculations

In the framework of the cellular analog of SFM in the idealized case with the homogeneous string distribution in the transverse plane ($\overline{k}_{ij} \equiv \eta = const$ for all i and j) the asymptotic of p_t-p_t correlation coefficient can be found analytically at large string density $\rho^{str} = \eta/\sigma_0$, $\eta \gg 1$, when one cluster is forming:

$$b_{p_t-p_t} = \frac{\mu_{0F}}{\mu_{0F} + 16\gamma^2 \sqrt{\eta}} \qquad \text{and} \qquad b_{p_t^2-p_t^2} = \frac{\mu_{0F}}{\mu_{0F} + 4\widetilde{\gamma}^2 \sqrt{\eta}} \tag{6}$$

Here μ_{0F} is the average number of charged particles produced from a decay of one string in the forward rapidity window. The γ is the coefficient of proportionality between the average transverse momentum $\overline{p}$ and the square root from the dispersion of p for one string: $\sigma_p = \gamma\overline{p}$, where $\sigma_p^2 = \overline{p^2} - \overline{p}^2$. The $\widetilde{\gamma}$ is the same for p^2: $\sigma_{p^2} = \widetilde{\gamma}\overline{p^2}$. In the last formula of (6) as the dynamical variables in the forward (F) and backward (B) rapidity windows instead of p_{tF} and p_{tB} (4) we have used:

$$p_{tF}^2 \equiv \frac{1}{n_F} \sum_{i=1}^{n_F} p_{tiF}^2, \qquad \text{and} \qquad p_{tB}^2 \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} p_{tiB}^2, \tag{7}$$

The asymptotics (6) are similar to the ones obtained in [11] for n-n and p_t-n correlations:

$$b_{n-n} = b_{p_t^2-n} = \frac{\mu_{0F}}{\mu_{0F} + 4\sqrt{\eta}} \qquad \text{and} \qquad b_{p_t-n} = \frac{1}{2} \frac{\mu_{0F}}{\mu_{0F} + 4\sqrt{\eta}} , \tag{8}$$

notwithstanding the fact that the derivation of (6) is very different and much more complicated. We see that for p_t-p_t correlation in asymptotic the $\mu_0/\sqrt{\eta}$-scaling obtained in [11] for n-n and p_t-n correlations is also takes place. The correlation coefficients depends only on the one combination of μ_0 and η.
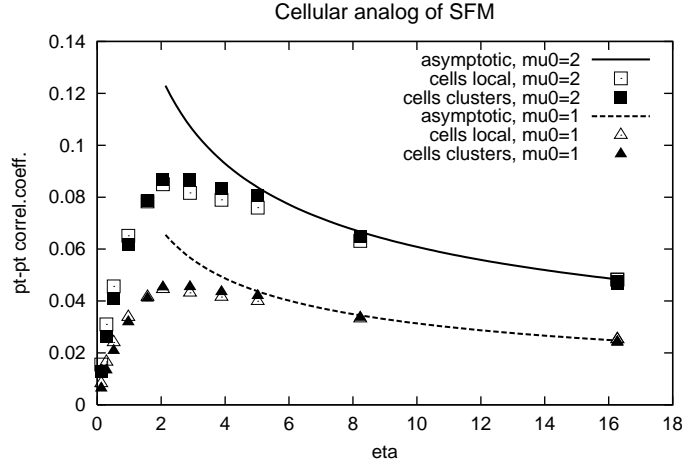

Figure 1: The comparison of the asymptotic of p_t-p_t correlation coefficient $b_{p_t-p_t}$ (6) with the results of its MC calculations in the framework of the cellular analog of SFM in the idealized case with the homogeneous string distribution in the transverse plane at two values of $\mu_{0F} \equiv \mu_0$. The η is proportional to the string density $\eta = \sigma_0 \rho^{str}$. The total number of sells is $M = 450$. See the text for details.
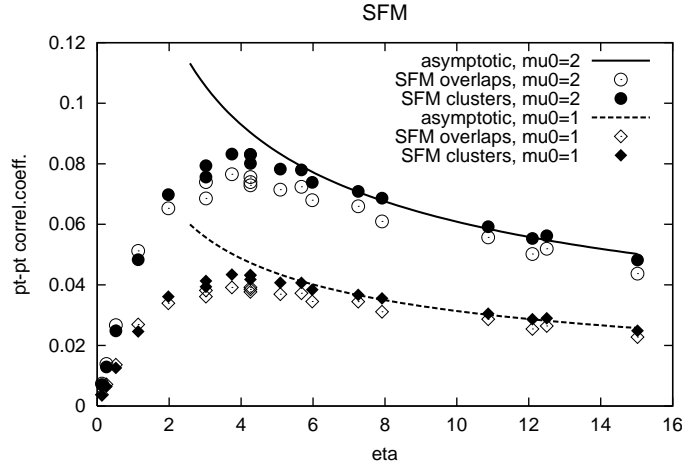

Figure 2: The same as in Fig.1 but for MC calculations in the framework of the original SFM at the average number of strings $N^{str} =$ 1000, 4000 and 8000.

In Fig.1 we compare the asymptotic (6) of $b_{p_t-p_t}$ with the results of its MC calculations in the framework of the cellular analog of SFM described above. We see that at $\eta > 5$ the results for both *local* and *global* versions of the model practically coincide with the asymptotic. The MC calculations in Fig.1 fulfilled for the total number of cells $M = 450$, but the simulations with the small number of cells $M = 45$ give practically the same results. So for p_t-p_t correlation one also has "M-scaling", which was found earlier in [11] for p_t-n and n-n correlations. This means that the p_t-p_t correlation coefficient depends only on the string density $\rho^{str} = \eta/\sigma_0$ but not on the total number of strings $N^{str} = \eta M$.

In Fig.2 we compare the same asymptotic (6) with the results of MC calculations in the framework of the original SFM (with the taking into account the real geometry of merging strings, which takes much more efforts). Again we see that at $\eta > 5$ the MC results calculated for *local (overlaps)* and *global*

(clusters) versions of the original SFM practically coincide with the asymptotic (6). In Fig.2 the points correspond to the MC calculations fulfilled at the different number of strings N^{str} =1000, 4000 and 8000. We see that in this case the results also do not depend on the total number of strings but depend only on the string density $\rho^{str} = \eta/\sigma_0$.

We would like to emphasize that this takes place only due to the fact that as observables p_{tF} and p_{tB} we choose the event mean values of transverse momenta of the particles emitted in the rapidity intervals (see (4) and (7)). In paper [14] it was shown that if one takes as observables the transverse momenta of single particles produced in these two rapidity windows (two particle correlation) then such defined p_t–p_t correlation coefficient is inversely proportional to the number of strings N^{str} and rapidly decreases to zero with the increase of the total number of strings.

Comparing the Fig.1 and Fig.2 we see also that the MC calculations in the framework of the original SFM and in the framework of its cellular analog give very similar results at all values of string density.

5 The p_t-p_t correlation in minimum bias AA-collisions

In the case with the realistic nucleon distribution density of colliding nuclei the results of the MC calculations of the p_t–p_t correlation function (see section 3): $\langle p_{tB} \rangle_{p_{tF}} = f(p_{tF})$ for minimum bias nuclear collisions at SPS, RHIC and LHC energies are presented in Fig.3. Certainly, in the case of minimum bias MC calculations one has to include the impact parameter b into definition of configuration C (see section 2). In these figures ($\circ$) and ($\bullet$) denote the results of calculations in the framework of the original SFM for its *local (overlaps)* and *global (clusters)* versions correspondingly. The ($\square$) and ($\blacksquare$) denote the results of calculations in the framework of the cellular analog of SFM for its *local* and *global (clusters)* versions. All presented results are for the forward rapidity window of 2 unit length ($\Delta y_F = 2$).

First of all we see in Fig.3 that the results of MC calculations for all four versions are practically coincide with each other. Only the results for both versions with local fusion are slightly shifted to the higher values of p_t in comparison with the ones for clusters, which can be easily explained by simple general arguments.

We see also that in the case of minimum bias nuclear collisions the p_t–p_t correlation functions $\langle p_{tB} \rangle_{p_{tF}}$ exhibit strongly nonlinear behavior. Note that the correlation functions calculated at fixed value of impact parameter (or with its small fluctuations within a certain centrality class) are practically linear and the strength of the correlation can be described by one number - the correlation coefficient (see (5)). It's not true for minimum bias collisions and the correlation can't be described by one number in this case - the derivative of the correlation function is positive at small values of p_{tF} and it becomes negative at large values of p_{tF}.

This can be explained in the following way. The set of minimum bias collisions includes both central and peripheral collisions. In our model with string fusion due to higher density of strings one has higher value of p_{tB} and p_{tF} for central collisions, than for peripheral ones. This is the reason for the positive p_t–p_t correlation. To understand the reason for the negative p_t–p_t correlation at large values of p_{tF} one has to remember that as observables p_{tF} and p_{tB} (4) (7) we choose the event mean values of transverse momenta of the particles emitted in the rapidity intervals. Only in this case the p_t–p_t correlation is not equal to zero in real heavy ion collisions (see the discursion in section 4). It's clear from (4) (7) that although the event mean values of p_{tF} and p_{tB} are higher for central collisions, their dispersions is higher for peripheral ones, as the number of charged particles n_F and n_B produced in these rapidity intervals are smaller for peripheral collisions. This means that if in some event one observes a very high (or a very low) value of p_{tF} in the forward rapidity window (than the typical value $\langle p_{tF} \rangle$), then it's more probably that the peripheral collision has taken place and then one finds the p_{tB} in the backward rapidity window at a low level - typical for peripheral collisions. So with the increase of p_{tF} the function $\langle p_{tB} \rangle_{p_{tF}}$ starts and finishes at a low (typical for peripheral collisions) value of p_{tB} obtaining in the middle more higher (typical for central collisions) value.

Using these arguments and taking into account the increase of string density, typical values of p_{tF} and p_{tB}, and the multiplicities n_F and n_B, with the increase of the initial energy, one enables, also to explain the change of the form of the p_t–p_t minimum bias correlation function in Fig.3 from SPS to RHIC and LHC energies.

6 Conclusion

The theoretical description of the correlations between observables in two separated rapidity intervals in relativistic nuclear collisions is presented. The calculations are fulfilled in the framework of the model taking into account the possibility of quark-gluon string fusion and also in the framework of the suggested simple cellular analog of this model. For both models two possible mechanisms of a string merging are considered: the local fusion ("overlaps") and the global fusion (with forming of colour "clusters").

It is shown, that for the p_t–p_t correlation the event-by-event correlation between event mean values of transverse momenta of the particles emitted in two different rapidity intervals does not decrease to zero with the increase of the number of strings in contrast with the correlation between the transverse momenta of single particles produced in these two rapidity windows which was studied earlier [14].

In the idealized case with the homogeneous string distribution in the transverse plane the asymptotic of p_t–p_t correlation coefficient analytically calculated in the framework of global string fusion mechanisms, when at large string density one cluster is forming. The asymptotic is compared with the results of the MC calculations in this idealized case both in the framework of the original SFM and in the framework of its cellular analog, which enables to control the MC algorithms.

In the case with the realistic nucleon distribution density of colliding nuclei the results of the MC calculations of the p_t–p_t correlation function for minimum bias nuclear collisions at SPS, RHIC and LHC energies are presented and analysed.

Acknowledgments

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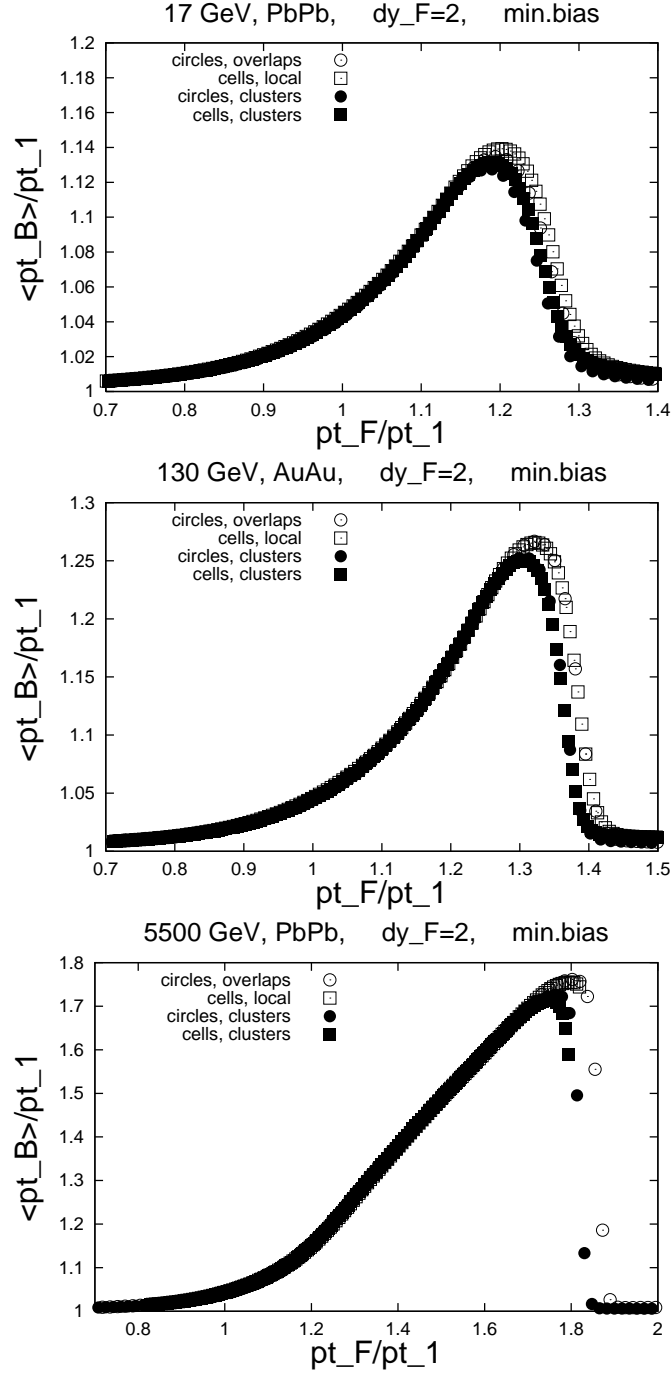   Eur. Phys. J. **C32** (2004) 535

Figure 3: The results of the MC calculations of the p_t–p_t correlation function $\langle p_{tB} \rangle_{p_{tF}} = f(p_{tF})$ for minimum bias nuclear collisions at SPS, RHIC and LHC energies. See the text for details. The results are presented in units of pt_1, where $pt_1 \equiv \overline{p}$ is the average transverse momentum of particles produced from a decay of one string.